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## OCA PAD AMENDMENT - PROJECT HEADER INFORMATION

11/02/93

Active

Project #: E-24-626                      Cost share #: E-24-321                      Rev #: 11  
Center # : 10/24-6-R7081-0A0              Center shr #: 10/22-1-F7081-0A0              OCA file #:  
Contract#: AFOSR-91-0013                      Mod #: AMENDMENT C                      Work type : RES  
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Contract entity: GTRC  
  
Subprojects ? : Y    CFDA: 12.800  
Main project #:    PE #:

Project unit:                      ISYE                      Unit code: 02.010.124  
Project director(s):  
SERFOZO R F                      ISYE                      (404)894-2305

Sponsor/division names: AIR FORCE                      / BOLLING AFB, DC  
Sponsor/division codes: 104                      / 001

Award period:              901101              to              931231 (performance)              940630 (reports)

Sponsor amount	New this change	Total to date
Contract value	0.00	151,319.00
Funded	0.00	151,319.00
Cost sharing amount		6,445.00

Does subcontracting plan apply ? : N

Title: STOCHASTIC NETWORK PROCESSES

## PROJECT ADMINISTRATION DATA

OCA contact: Anita D. Rowland                      894-4820

Sponsor technical contact                      Sponsor issuing office

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AFOSR/NM                      AFOSR/PKA  
BUILDING 410                      BUILDING 410  
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Defense priority rating : N/A                      supplemental sheet  
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Administrative comments -

AMENDMENT C EXTENDS THE PERIOD OF PERFORMANCE AND REPORT SCHEDULE.

GEORGIA INSTITUTE OF TECHNOLOGY  
OFFICE OF CONTRACT ADMINISTRATION

NOTICE OF PROJECT CLOSEOUT

Closeout Notice Date 04/01/94

Project No. E-24-626\_\_\_\_\_ Center No. 10/24-6-R7081-0A0\_

Project Director SERFOZO R F\_\_\_\_\_ School/Lab ISYE\_\_\_\_\_

Sponsor AIR FORCE/BOLLING AFB, DC\_\_\_\_\_

Contract/Grant No. AFOSR-91-0013\_\_\_\_\_ Contract Entity GTRC

Prime Contract No. \_\_\_\_\_

Title STOCHASTIC NETWORK PROCESSES\_\_\_\_\_

Effective Completion Date 931231 (Performance) 940630 (Reports)

Closeout Actions Required:	Y/N	Date Submitted
Final Invoice or Copy of Final Invoice	Y	_____
Final Report of Inventions and/or Subcontracts	Y	_____
Government Property Inventory & Related Certificate	N	_____
Classified Material Certificate	N	_____
Release and Assignment	Y	_____
Other _____	N	_____
Comments _____		

Subproject Under Main Project No. \_\_\_\_\_

Continues Project No. \_\_\_\_\_

Distribution Required:

Project Director	Y
Administrative Network Representative	Y
GTRI Accounting/Grants and Contracts	Y
Procurement/Supply Services	Y
Research Property Management	Y
Research Security Services	N
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NOTE: Final Patent Questionnaire sent to PDPI.

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19. ABSTRACT (Continue on reverse if necessary and identify by block number) This report summarizes the research done on AFOSR Grant 89-0407 during the last year. The highlights are a summary of work on "Travel and Sojourn Times in Stochastic Networks", "Queueing Networks with Dependent Nodes and Concurrent Movements" and "Optimality of Routing and Servicing in Dependent Parallel Processing Systems".					
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January 22, 1992

Dr. Jon A. Sjogren  
AFOSR/NM Bldg 410  
Bolling AFB, DC 20332-6446

Dear Jon:

Enclosed is the final report on my last one-year grant with AFOSR. I recently sent you two papers documenting part of our research (a reprint of a published paper and a preprint of a paper submitted for publication).

Best Regards,

~

Richard F. Serfozo



## Final Technical Report

Project Title: Stochastic Network Processes

Contract No: AFOSR 91-0013

Time Period Covered: Oct 1, 1990 to Oct 31, 1991

Principle Director:

Richard F. Serfozo, Professor

School of Industrial and Systems Engineering

Atlanta, GA 30332-0205

# Stochastic Network Processes

## 1 Introduction

This report summarizes the highlights of our research accomplishments during the last year. The general theme of our research has been the development of stochastic network processes for modeling manufacturing, computer and telecommunications networks. Our emphasis during the last year has been on travel times in networks, networks with dependent nodes and current movements, and resource allocation in parallel processing systems. The following is a sketch of the documentation of our results in these topic areas.

## 2 Travel Times in Stochastic Networks

Our major paper on this topic is:

Kook, K. H. and R. F. Serfozo (1991) "Travel and Sojourn Times in Stochastic Networks". To appear in *Ann. of Applied Probability*.

A precursor of this paper was a paper in the *Proceedings of the Fifth GI-ITG Conference on Measurement, Modeling and Evaluation of Computer Systems and Networks* (the plenary address of the meeting). Another paper on this subject is under preparation.

Travel times in networks are a major performance characteristic. For the classical open Jackson network it was known that the travel time on an "overtake-free" route is the sum of independent exponentially distributed delays at the nodes on the route. This result was proved by Reich (1957) for queues in tandem and by Walrand and Varaiya (1980) for the open network. Kelly and Pollett (1983) proved an analogous result for a closed Jackson network. The simply-stated problem of finding the mean travel time to go from one sector (set of nodes) of the network to another was thought to be formidable. This was the problem that motivated our research on more general travel times that include the following:

- The time a unit spends in a sector during its stay in a network.
- The time a unit spends as a certain type between two visits to a node.
- The time for a unit to move through a series of sectors.

Two of our main results are an expression for the expectation of a wide class of travel times in networks and a related law of large numbers. The proofs of these did not follow as readily as other network results have, because one has to look into the “future” of the network to analyze such travel times. Our other main result is a characterization of the travel times on overtake-free routes in networks with dependent nodes. The novelty of this was the use of Palm probabilities to cut through the notational barrier that one faces with network analysis.

### **3 Stochastic Network Processes with Dependent Nodes and Concurrent Movements**

Our papers in this area are:

Serfozo, R. F. (1991) “Reversibility and Compound Birth-Death and Migration Processes”. To appear in *Queueing Processes and Related Models*, Oxford Univ. Press.

Serfozo, R. F. (1991) “Queueing Networks with Dependent Nodes and Concurrent Movements”. Submitted for publication.

The classical birth-death process describes a wide class of service systems with queueing. In studying systems with concurrent (compound, synchronous or batch) movements, we were surprised that a birth-death process for group births and deaths and multi-type units had not been developed. Apparently researchers tended to think in terms of only one unit moving at a time and that concurrent movements were too messy. We went ahead and developed such a birth-death process which we call a compound birth-death process. In doing so, we found some new characterizations of reversible Markov processes. We also developed and reversible network processes with concurrent movements, which we call compound migration processes.

We gained new insights on networks with dependent nodes. This culminated in developing a “basic” network process that includes the classical Jackson, BCMP, Kelly, and Kelly-Whittle networks. We also developed several variants of the basic network process for modeling networks with interacting subpopulations and certain types of concurrent movements. We also uncovered a relation between weakly coupled

networks and quasi-reversible networks and the classical networks that simplifies the theory of these processes and their connection with the rest of the theory.

#### **4 Resource Allocation in Parallel Processing Networks**

Our work on this is documented in the following article and PhD dissertation:

Menich, R. P. and R. F. Serfozo (1991) "Optimality of Routing and Servicing in Dependent Parallel Processing Systems". *Queueing Systems*, **9** 403-418.

Menich, R. P. (1991) *Resource Allocation in Parallel Processing Networks*. PhD Dissertation at Georgia Tech.

The main issues we addressed for a system of dependent, parallel processing systems are: When is it optimal to route customers to the shortest queue? When is it optimal to devote auxiliary service capacity to serve the longest queue? We proved that these RSQ and SLQ policies are optimal for a wide class of dependent Markovian systems that satisfy certain symmetry conditions.

The second part of the dissertation addresses the problem of determining resource levels (numbers of cutting tools, pallets, material, etc.) to support a Flexible Manufacturing Systems. The objective is to maximize the throughput of parts subject to a desired relative production ratios of parts. Using some stochastic analysis and linear programming, we obtained a relatively simple-looking formula for the resource levels.

E24-626 2

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# Status Report on AFOSR Project 91-0013 Stochastic Network Processes

Richard F. Serfozo  
Georgia Institute of Technology

Dec 10, 1992

The general theme of our research has been to develop stochastic network processes for modeling the movement of discrete units in networks. Primary examples are the movement of parts and supplies in manufacturing plants and in distribution systems and the movement of data packets and telephone calls in computer and telecommunications networks. The distinguishing feature of our research is the emphasis on the next generation of *intelligent* networks that will be the backbone of our manufacturing and computer systems. In these networks, the processing of units at the nodes and the routing of units typically depend dynamically on the actual network congestion, and units move concurrently (e.g. batch processing). Most of the present theory of stochastic network processes is for unintelligent networks in which the nodes operate independently, the routes of units are independent, and the units move one-at-a-time. Our goal is to provide an understanding of these more complex intelligent networks by describing their stochastic behavior. The following is a summary of the papers we have written for this project during the last two years.

**1 Optimal Routing and Servicing in Dependent, Parallel Processing Systems** by R. Menich and the PI. *Queueing Systems*, 9, 403-418, 1991. In a system of independent, identical parallel processing service stations, customers are typically routed to the shortest queue. And any auxiliary mobile server is typically assigned to serve the longest queue. When are these

dynamic control policies optimal for *dependent* parallel processing stations? We show they are optimal for a wide class of Markovian systems in which the arrival and service rates at the stations (which may depend on the numbers of customers at *all* the stations) satisfy certain symmetry and monotonicity conditions. Under these policies the queue lengths will be stochastically smaller than the queue lengths under any other policy. Furthermore, these policies minimize standard discounted and average cost functions over finite and infinite horizons.

**2 Reversibility and Compound Birth-Death and Migration Processes** by the PI. In *Queueing and Related Models*, Oxford University Press, 1992.

The classical birth-death stochastic process is one of the most useful processes for modeling a variety of service or input-output systems with queueing. This process represents a single population and units arrive and depart one-at-a-time. What would be the analogous process for modeling several dependent populations of units, where the units may arrive and depart in batches? We have developed such a process called a multi-variate, compound birth-death process. In doing so, we have solved some basic problems on the characterization of reversible stochastic processes whose trajectories are generated by certain basic vectors. We have also developed an analogous network process called a compound migration process. Here the populations interact and units leaving a population may move to other populations or exit the network singly or in batches. We identify several families of these processes that are especially suited for manufacturing and communications networks and describe their equilibrium behavior and other performance parameters.

**3 Queueing Network Processes with Dependent Nodes and Concurrent Movements** by the PI. To appear in *Queueing Systems*, 1992. This work describes a new class of network processes for modeling networks with dependent nodes and concurrent movement of units. It also shows the structural relations between the classical Jackson network and its ostensibly different variations (e.g. quasi-reversible and weakly coupled networks); these are all special cases of a certain "basic" partially balanced network process. This insight provides a general framework for representing some partially balanced dependencies relating to the following situations:

- Alternate routing of units to avoid congestion.

- The arrival rate of units into a network depends on the number of units already there.
- Units are processed in parallel at several nodes.
- The service rate at a node depends on the states of the other nodes (e.g. the rate decreases as downstream congestion decreases.)
- A “starved” node with few units pulls units from its upstream neighbors.
- Service rates are dynamically regulated to increase throughput or balance the workload at nodes.
- Several units merge into one (e.g. manufacturing assemblies) or one unit splits into many (e.g. one email message propagates a family of messages).
- A unit moving in a network requires shared resources (e.g. tools, tokens, data files) that move concurrently with it.

In addition to describing the equilibrium behavior, we describe Monte Carlo methods for computing some of the network performance parameters for large networks.

**4 Travel and Sojourn Times in Stochastic Networks** by K. Kook and the PI. To appear in *Annals of Applied Probability*, 1993.

Ever since the Jackson network process was developed about 30 years ago, a major concern has been how long does it take a unit to travel through a network? Essentially the only basic result in this regard, aside from approximations, is that in an open Jackson network without overtaking (a very restrictive condition), the travel time on a route is the sum of independent exponential sojourn times at the nodes on the route. An analogous result holds for a closed Jackson network (but the node sojourn times are no longer exponential). A thirty year old problem for the Jackson network had been “What is the expected time to travel from one node (or sector of nodes) to another? We solved this and related problems for Jackson networks and for other networks with congestion dependent routing and processing. In doing this, we addressed a large class of travel times in networks (e.g. the time to



visit a node 3 times, or the time to visit each node at least once) and derived an expression for their expected values. We also, extended the classical results for characterizing the distribution of the travel time on overtake free routes for the more general networks with dependent nodes.

**5 Travel Times in Queueing Networks and Network Sojourns** by the PI. Submitted for publication.

This paper contains new Little laws for networks with dependencies. It also reviews what is known about travel times of units in a network. We present expressions for sojourn times such as the time it takes a unit to move through a network, or the time a unit spends in a sector of a network as a certain type of unit before changing its type and entering another sector. Other problems we address are finding the expected length of a busy period of a node, and finding the expected time interval that the number of units at a node exceed a certain high level. Such information on “high-level” exceedances or extreme values of queueing networks is important for predicting extraordinary work loads (overtime) or emergency situations.

**6 Relating the Waiting Times and Queue lengths in Heavy Traffic Systems** by W. Szczotka, W. Topolski and the PI. Submitted for publication.

For a single-server queueing system, intuition suggests that an arriving unit's waiting time before its service begins is approximately equal to the number of units in the system times the average service time. Why are there no results to this effect? Because it is not true when the traffic intensity is moderate. However, we show that this approximation is indeed valid for certain systems in heavy traffic. We also give more insight into diffusion approximations for queueing systems. Namely, there are three types of natural diffusion approximations, not just the conventional reflected Brownian motion model. This analysis is a precursor to results for networks.

**7 Extreme Values of Random Times in Stochastic Networks** by Sungyeol Kang. A PhD dissertation in Industrial and Systems Engineering, Georgia Tech.

In manufacturing systems, a typical concern is the time it takes for a group of units that will eventually constitute one system to be processed by a network of work stations. This time is the maximum (or extreme value) of the travel times of the units through the network. An example is that 20 units must pass

through a PERT network before they are brought together as one system. The individual network completion times often have distributions that are of phase type or are mixtures of such distributions. This dissertation is a study of extreme values of such distributions. We show that the extreme value distribution of a large sample is asymptotically a Gumbel distribution. We apply these results to model completion times in a Markov PERT network and for group travel times in open Jackson networks when the traffic is light or when the traffic is heavy and the special units are interspersed evenly among many other units.

**8 Extreme Queues and Stationarity of Service Systems in Heavy Traffic** by Kuo-Hwa Chang. A PhD dissertation in Industrial and Systems Engineering, Georgia Tech.

Knowledge of extreme queue lengths in service systems is needed for designing adequate space in manufacturing or communication systems or predicting when capacity constraints are violated. We show that the distribution of the maximum queue length in a time interval for a queueing system in heavy traffic converges to a new type of extreme value distribution. We also study the processes that record the number of times that the queue length exceeds a high level and the cumulative time the queue is above the level. We show that these processes converge in distribution to compound Poisson processes. The limiting extreme value distribution and compound Poisson processes we obtained can be used in practical computations similarly to the use of limiting normal distributions in central limit phenomena.

The second part of this study concerns a tree-like service system. These networks typically have dependencies that are not amenable to a Markovian analysis. An example of this is a network in which a unit requires the same service time at each node it visits. Furthermore, the service times and routing of the units may depend on each other. We study a tree-like network with very general non-Markovian dependency assumptions on the interarrival times, service times and routes. We give conditions under which the system is stable (the queue length does not tend to infinity). We show that the waiting times and queue lengths are stationary or asymptotically stationary under some natural conditions, and we characterize the limiting distribution of the system. Finally, we derive the limiting distribution of the waiting times when the traffic in the network is heavy.

Final Technical Report  
on  
AFOSR Project 91-0093  
Stochastic Network Processes  
February 27, 1994

Principal Investigator: Richard F. Serfozo, Professor

School of Industrial and Systems Engineering

Georgia Institute of Technology

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(404) 894-2305

The general theme of our research has been to develop stochastic network processes for modeling the movement of discrete units in networks. Primary examples are the movement of parts and supplies in manufacturing plants and in distribution systems and the movement of data packets and telephone calls in computer and telecommunications networks. The distinguishing feature of our research is the emphasis on the next generation of *intelligent* networks that will be the backbone of our manufacturing and computer systems. In these networks, the processing of units at the nodes and the routing of units typically depend dynamically on the actual network congestion, and units move concurrently (e.g. batch processing). Most of the present theory of stochastic network processes is for unintelligent networks in which the nodes operate independently, the routes of units are independent, and the units move one-at-a-time. Our goal is to provide an understanding of these more complex intelligent networks by describing their stochastic behavior. The following is a summary of the papers we have written for this project during the last year.

**1 Relating the Waiting Times and Queue lengths in Heavy Traffic Systems** by W. Szczotka, W. Topolski and the PI. To appear in *Stochastic Processes Appl.* 1994.

For a single-server queueing system, intuition suggests that an arriving unit's waiting time before its service begins is approximately equal to the number of units in the system times the average service time. Why are there no results to this effect? Because it is not true when the traffic intensity is moderate. However, we show that this approximation is indeed valid for certain systems in heavy traffic. We also give more insight into diffusion approximations for queueing systems. Namely, there are three types of natural diffusion approximations, not just the conventional reflected Brownian motion model. The results apply to rather general non-standard queueing systems in which the services may be highly dependent on the arrivals. We are currently writing another paper with a similar theme, involving more intricate analysis, for tree-like stochastic networks.

**2 Extreme Queues and Stationarity of Service Systems in Heavy Traffic** by Kuo-Hwa Chang. PhD dissertation in Industrial and Systems Engineering, Georgia Tech, 1993.

Knowledge of extreme queue lengths in service systems is needed for designing adequate space in manufacturing or communication systems or predicting when capacity constraints are violated. We show that the distribution of the maximum queue length in a time interval for a queueing system in heavy traffic converges to a new type of extreme value distribution. We also study the processes that record the number of times that the queue length exceeds a high level and the cumulative time the queue is above the level. We show that these processes converge in distribution to compound Poisson processes. The limiting extreme value distribution and compound Poisson processes we obtained can be used in practical computations similarly to the use of limiting normal distributions in central limit phenomena. We are currently writing two papers based on the results in this dissertation.

**3 Little Laws for Waiting Times and Utility Processes** by the PI. Submitted for publication, 1993.

Little's law for queueing systems is  $L = \lambda W$ : the average queue length equals the average arrival rate times the average waiting time in the system. Although there is considerable knowledge about this law, its applicability for many systems is still an open question. This study gives further insights into techniques for establishing such laws for new systems, and it presents several basic laws for systems with special structures. The main results concern (1) general necessary and sufficient conditions for Little laws for utility processes as well as queueing systems, (2) Little laws for systems that empty out periodically or, more generally, have regular departures (3) Little laws tailored to regenerative, Markovian and stationary systems and Little laws for stochastic networks. The motivation for a revisit to this subject was that the existing theory did not apply directly to stochastic network processes and further work was needed to obtain Little laws for networks.

**4 Performance Analysis and Improvement of Parallel Simulation** by Liang Chen. PhD dissertation in Industrial and Systems Engineering, Georgia Tech, 1993.

This covers the following topics:

- (1) Comparison of several conservative parallel-simulation protocols with the Time Warp protocol.
- (2) A Markovian model for analyzing the effect of memory capacity on Time Warp performance.
- (3) Time Warp analysis for queueing network simulations.
- (4) Space-Time Division protocols for feedforward parallel-simulations.

**5 The Effect of Memory Capacity on Time Warp Performance** by Akylidiz, I.F. Chen, L., Das, S.R., Fujimoto, R.M. and the PI. J. Parallel and Distributed Computing, 18, 411-422.

This is a study of a parallel simulation in which several interacting processes are synchronized by the *Time Warp* protocol and a "cancelback" scheme is used to reclaim storage when the system runs out of memory. A Markov process model is developed for describing the behavior of the simulation. Namely, it shows how the speedup of the system changes as the amount of memory is varied. The model is validated through performance measurements on a Time Warp system executing on a shared-memory multiprocessor using a workload similar to that in the model. It is observed that if the sequential simulation requires  $m$  message buffers, Time Warp with a small fraction of message buffers beyond  $m$  performs almost as well as Time Warp with unlimited memory.

**6 Bounds on Speeds of Parallel Simulations That Satisfy a Conservation Principle** by Chen L. and the PI. Submitted for publication, 1993.

Many parallel simulations have natural constraints on the number of processors that may usefully be employed. We provide insight into this constraint by deriving upper bounds on the simulation speed (the rate at which it processes real events) for three processor-assignment schemes: fixed, global and local processor assignments. Our analysis is based on representing a parallel simulation as a random time transformation of the system being simulated. Using a sample path approach for stochastic processes, we show that a broad class of parallel simulations satisfy a conservation principle: The long run average virtual times in the simulation are equal (otherwise the virtual times will diverge from each other). We use this principle to derive the bounds and compare the efficiencies of the strategies.

**7 Parallel Simulation by Multi-Instruction, Longest-path Algorithms** by Chen, L. and the PI. Submitted for publication, 1993.

This paper presents several basic algorithms for the parallel simulation of G/G/1 queueing systems and certain networks of such systems. The coverage includes systems subject to manufacturing or communication blocking, or to loss of customers due to capacity constraints. The key idea is that the customer departure times are represented by longest-path distances in directed graphs instead of by the usual recursive equations. This representation leads to algorithms with a high degree of parallelism that can be implemented on parallel computers with single or multiple instruction streams.

**8 Completion Times of Parallel Task Graphs: Extreme Values of Phase-type and Mized Random Variables** by Sungyeol Kang and the PI. A paper in preparation.

In manufacturing systems, a typical concern is the time it takes for a group of units that will eventually constitute one system to be processed by a network of work stations. This time is the maximum (or extreme value) of the travel times of the units through the network. An example is that 20 units must pass through a PERT network (or a task graph) before they are brought together as one system. The individual network completion times often have distributions that are of phase type or are mixtures of such distributions. This paper is a study of extreme values of such distributions. We show that the extreme value distribution of a large sample is asymptotically a Gumbel distribution. We apply these results to model completion times in a Markov task graphs and for group travel times in related networks.